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THERMAL DIAGRAMS AND THEIR PRACTICAL USE.

BY HUGO DIEMER.

THE subject of this discussion is not a new one, but it is believed that the manner of presentation may be found advantageous. The indicator diagram taken from piston engines reveals the relations between pressure and volume at all points of the cycle. The area which the diagram encloses measures, when properly scaled, the amount of work done. The indicator card, in connection with a dynamometer or brake test, affords complete data as to the mechanical efficiency of a heat engine. It does not reveal *prima facie* anything with regard to the thermodynamic efficiency of an engine, or of the heat interchanges taking place during the cycle. The total area underneath any line in an indicator diagram represents foot-pounds of work done by or on the medium during the change of condition represented by that line. It would be desirable to have an instrument which might record in a similar way, by areas underneath lines, the total heat rejected or taken in by the medium during the same cycle of operations shown by the indicator diagram. The areas in this case would measure thermal units, whereas in the indicator diagram they measure foot-pounds.

In a system representing work by areas, the two factors whose product make up the area are units of force and of distance—that is, units of intensity and of extent. In a system representing heat units by areas, we will at once agree that the factor of intensity must be temperature. To the factor of extent has been given the name entropy.

The plotting of thermal diagrams with temperature and entropy as coordinates is of great practical value in investigations of thermal efficiency of heat engines. It is not only briefer than analytic methods, but it is more completely exact, since it gives the conditions existing at all points, whereas analytic methods can only establish an average condition during a certain period.

The first task in preparing the heat diagram is to define mathematically the unit of heat extent, so that it may be deduced from known properties of the medium dealt with.

Taking first the case of perfect gases:

If heat is applied to a gas, the results of the transfer may be expressed mathematically by the equation—

$$dQ = c_v dT + Apdv, \quad (1)$$

where Q represents the heat supplied, c_v the specific heat at constant volume, dT the increase in temperature and p the pressure during the infinitesimal change in volume dv, and A the heat equivalent of work, or $\frac{1}{778}$. $c_v dT$ represents the heat involved in the temperature change, and $Apdv$ the heat equivalent of the external work.

The combined laws of Boyle and Charles for gases are expressed mathematically by the equation $pv = RT$, in which p is the pressure, v the volume, T the absolute temperature, and R a constant depending on the medium. From this equation we have:

$$p = \frac{RT}{v} \quad (2)$$

Substituting this value of p in equation (1), we have:

$$dQ = c_v dT + ART\frac{dv}{v} \quad (3)$$

By definition, entropy or ϕ is equal to $\int \frac{dQ}{T}$ Hence, dividing equation (3) by T, we have:

$$d\phi = \frac{dQ}{T} = c_v \frac{dT}{T} + AR\frac{dv}{v} \quad (4)$$

The heat absorbed at constant pressure is $c_p dT$, while that required to raise the temperature at constant volume is $c_v dT$. The difference $(c_p - c_v)dT$ must be equal to the amount of heat necessary to effect expansion at constant pressure, namely, $Apdv$, or

$$(c_p - c_v)dT = Ap\frac{dv}{dT}dT \quad (5)$$

$$(c_p - c_v) = Ap\frac{dv}{dT} \quad (6)$$

If we consider pressure to be constant, we may differentiate the equation $pv = RT$, writing it $pdv = RdT$, and substituting this value for pdv in equation (6), we have:

$$c_p - c_v = AR \quad (7)$$

Substituting this value for AR in equation (4), we have:

$$d\phi = c_v\frac{dT}{T} + (c_p - c_v)\frac{dv}{v} \quad (8)$$

The general law of expansion of a perfect gas may be expressed by the equation

$$p_1 v_1^n = p_2 v_2^n = pv^n \quad (9)$$

or, since $pv = RT$, we may write equation (9) as:

$$RTv^{n-1} = p_1 v_1^n \quad (10)$$

or $RTv^{n-1} - p_1v_1^n = 0$. Differentiating, we have:

$$(n-1)RTv^{n-2}dv = -Rv^{n-1}dT, \text{ or } dv = -\frac{Rv^{n-1}dT}{(n-1)RTv^{n-2}}$$

$$= -\frac{vdT}{(n-1)T}; \text{ or: } \frac{dv}{v} = -\frac{1}{n-1}\frac{dT}{T} \qquad (11)$$

Substituting this value of $\frac{dv}{v}$ in equation (8), we have:

$$d\phi = c_v\frac{dT}{T} + (c_p - c_v)\left\{-\frac{1}{n-1}\frac{dT}{T}\right\} = \frac{dT}{T}\left\{\frac{nc_v - c_v - c_p + c_v}{n-1}\right\}$$

$$= \frac{dT}{T}\left\{\frac{nc_v - c_p}{n-1}\right\} = c_v\left\{\frac{n - \frac{c_p}{c_v}}{n-1}\right\}\frac{dT}{T};$$

integrating between limits T_1 and T_2 for which the entropies are ϕ_1 and ϕ_2, we have:

$$\phi_1 - \phi_2 = c_v\left\{\frac{n - \frac{c_p}{c_v}}{n-1}\right\} log_e\frac{T_1}{T_2} \qquad (12)$$

Equation (12) gives a means for determining the linear length of the entropy factor. The temperatures T_1 and T_2 are disclosed by the pv conditions shown by the indicator card. The exponent n is found by taking any two points on the curve, such as p_1v_1 and p_2v_2. Since we have $p_1v_1^n = p_2v_2^n$, we have, also, $\left\{\frac{p_2}{p_1}\right\} = \left\{\frac{v_1}{v_2}\right\}^n$ or, taking their logarithms, $log\ p_2 - log\ p_1 = n(log\ v_1 - log\ v_2)$. Hence,

$$n = \frac{log\ p_2 - log\ p_1}{log\ v_1 - log\ v_2}, \text{ or, } n = \frac{log\ p_1 - log\ p_2}{log\ v_2 - log\ v_1}$$

For accurate work, a number of points should be taken, the method of least squares being used if the requirements of the calculation demand it.

In the case of vapors such as steam, ether, alcohol, etc., the general case is that in which a part x of original liquid has been converted into vapor, the portion $1-x$ remaining liquid. If a unit weight be considered as raised from the freezing-point to a temperature T_1 and the part x is evaporated at that temperature, the increase in entropy takes place in two stages, that of the liquid part and that of the vapor part.

If the heat contained in the liquid part be denoted by q, then the entropy of the liquid,

$$\frac{dq}{T} = \int_{T_{32}}^{T}\frac{dq}{T} = c_v\int_{T_{32}}^{T}\frac{dT}{T} = c_v\ log_e\frac{T}{T_{32}}$$

Owing to the differing of the specific heat at different temperatures, this value for entropy of the liquid should be built up of small values corresponding to small increments of temperature, using the value of the specific heat corresponding to the temperature range taken.

In order to determine the entropy of the vapor portion, if r represents the latent heat of vaporization at the temperature at which

vaporization takes place, the entropy of vaporization will be $\frac{xr}{T}$, and the total entropy of the liquid and vapor will be

$$\frac{xr}{T} + c_v \log_e \frac{T}{T_{32}}$$

To show the connection between the entropy diagram and the steam-engine-indicator diagram, it is best to consider the thermodynamic action of the whole system — boiler, engine, condenser, and feed-water heater. Although the consecutive operations for turning heat into work take place in different organs of the system, these consecutive operations may be considered as the successive phases of a closed cycle in which we have (see fig. 1), first, the line *ab*, representing evaporation of the liquid in the boiler, at temperature *T*, into steam of quality of dryness *x*. Work is done during this process at constant pressure on the piston. The volume of the pound of water will

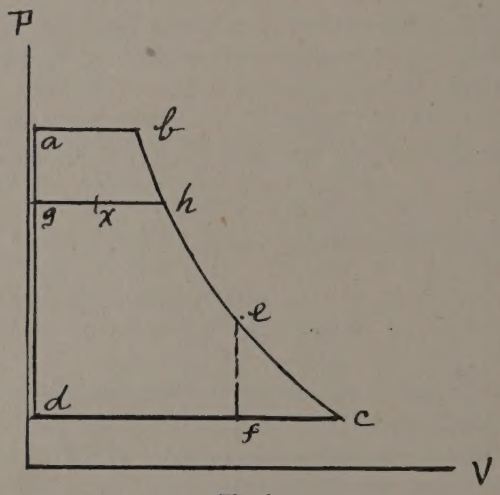

Fig. 1.

not vary very much from .017 cubic feet; *b* represents the volume of the pound of liquid after it has been expanded into steam. Assuming that evaporation has taken place, suppose the steam to be allowed to expand in such a manner that at all pressures it remains dry; that is, none of it condenses, and it is not superheated. We may plot these volumes at various pressures by inserting the known specific volumes of steam. The line *bc* is thus plotted. The next process in the system as a whole is the behavior, after expansion, of this one pound of steam at temperature 212° F. and atmospheric pressure. This step is condensation into feed water. This condensation occurs at constant pressure of the atmosphere, and will consequently be represented by the constant pressure line *cd*, indicating a diminution in volume from about 26 cubic feet to about .017 cubic feet. The last stage is repre-

sented by the supplying of heat from the source of heat between temperatures T_2 and T_1, or 212° F., and the temperature corresponding to the initial pressure. The cycle of operations just described is what is known as the Rankine ideal steam-engine cycle, and has been adopted by the British Institute of Civil Engineers as the standard for comparison of thermodynamic efficiencies of steam-engines.

Let us now trace out the corresponding entropy diagram:

At state a we have one pound of water whose temperature is known. The entropy of the liquid, $c_v log_e \frac{T}{T_{32}}$, calculated by the increment method already referred to, may be found in tables prepared by Peabody and others.

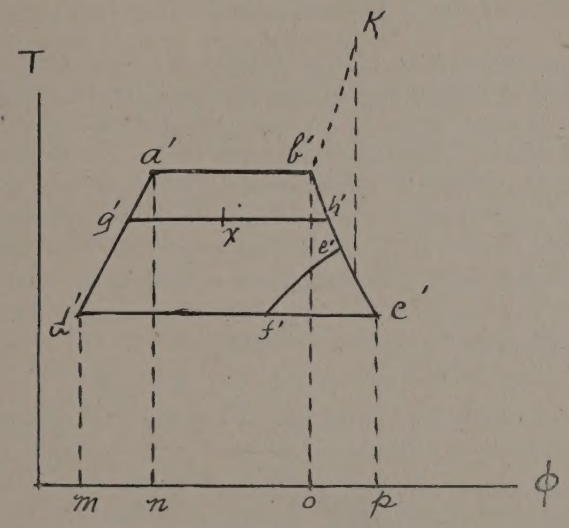

Fig. 2.

Plotting a suitable distance on the Y axis to represent the temperature, we plot a suitable distance to scale, horizontally to the right to represent the entropy of the liquid, found as above indicated, thus locating the point a' (fig. 2); b' is located by adding the entropy of vaporization, that is, $\frac{xr}{T}$, or, for complete evaporation, $\frac{r}{T}$. If the X axis has been taken as representing 32° F., the rectangle underneath $a'b'$ will represent to scale the latent heat of vaporization. This gives us the means for determining the area corresponding to one heat unit.

Points on the curve $b'c'$ are found by calculating the entropy values belonging to the respective temperatures corresponding to selected pressures; $c'd'$ represents abstraction of heat at constant temperature, and $d'a'$ represents supply of heat to the water in the boiler. The area $md'rn$ represents the heat required to heat the feed water from 32° F.

to the boiling-point; the area $md'a'n$ represents the heat required to heat the water to the point of vaporization; $na'b'o$ is the heat taken in during evaporation; $pc'd'm$ is the heat rejected during exhaust; $a'b'c'd'$ is the heat equivalent of the work done.

In actual engines, the expansion, instead of being carried to atmospheric pressure, will stop at some point such as e (fig. 1), by the opening of the exhaust, and there is a drop in pressure and temperature approaching a constant-volume line, ef. In order to plot ef on the $T\phi$ coordinates, we might find temperatures corresponding to slight drops in pressure, and find the corresponding entropy values from the formula:

$$\int \frac{dQ}{T} = c_v \log_e \frac{T}{T_2}.$$

The curve may also be plotted graphically by making use of the principle that corresponding points in each diagram divide the horizontal lines through them in the same proportion. For example, to transfer the point x from the pv diagram to the $T\phi$ diagram: Draw a horizontal gh through x, and draw also g^1h^1 at the temperature corresponding to the pressure at g. Divide g^1h^1 in x^1 in the same proportion as gh is divided in x. Then we know that the percentage of water evaporated, when x denotes its condition as to quality of dryness, will be in the same ratio to gh, denoting complete evaporation, as the heat required for partial evaporation will be to the heat required for complete evaporation; hence, $\frac{gx}{gh} = \frac{g^1x^1}{g^1h^1}$.

In order to apply this principle to transfer the constant-volume line ef to the entropy diagram, we need simply take a number of points on the line ef, pass horizontals through them, and draw the corresponding horizontals on the entropy diagram, dividing the latter set in the same proportion that the horizontals were divided in the pv diagram by the line ef.

It is evident that we may thus transfer any number of constant-volume and constant-pressure lines to the entropy diagram, and that, if a sufficient number are plotted, the diagram becomes a chart which can be prepared in form for blue-printing or other method of duplication, and that any points, p_1v_1, p_2v_2, etc., may be exactly transferred onto the chart by finding the intersection of the constant-volume and constant-pressure lines corresponding to p_1v_1, p_2v_2, etc., on the entropy chart.

As the chart represents the behavior of one pound of feed, it is necessary, however, to transform an actual indicator diagram into a diagram representing the behavior of one pound of steam before the transfer to the entropy chart is made. This is done by measuring the amount of cylinder feed by a water meter in the feed-water pipe, or

by using a graduated feed-water supply tank, and obtaining the number of strokes per pound of cylinder feed, in case the boiler supplies but the one engine, the necessary deduction being made for the feed pump. In case there are other steam-consuming organs besides the engine under test, the exhaust steam of the engine must be the gauge of the weight of cylinder feed.

A Rankine ideal engine cycle is next constructed for one pound of steam. The average indicator card of the engine is next transformed into a pv diagram for one pound of feed and superposed on the Rankine diagram, the relative clearance volume being added to the displacement volume shown by the indicator card. Or, the indicator card itself may be surrounded by the Rankine ideal cycle for the weight involved in the indicator diagram and a reduction factor employed in the transfer to the entropy chart.

Superheating effects are shown on the $T\phi$ diagram by making use of the fact that entropy of superheating must be equal to specific heat of superheated steam $\times log_e \frac{T_{sup}}{T_{sat}}$. By taking intermediate temperatures between the temperature of the superheating and that of saturation, the curve $b'k$ (fig. 2) is obtained. If, in transferring the indicator card to the entropy chart, it is found that the volume is greater than that of saturated steam, use must be made of the relation, experimentally determined, that exists between the pressure, volume and temperature of superheated steam, as expressed by Peabody, namely: $pv_{sup} = 93.5\ T_{sup} - 971\ p^{\frac{1}{4}}$, to determine the temperature of superheating.

Thermal diagrams constructed in accordance with the above methods, added to the information given by the indicator card and other data usually taken during an engine test, disclose completely all heat interchanges and heat relationships. The degree of refinement to which calculations based upon them can safely be carried depends necessarily upon the refinement and accuracy of the observational data from which they have been built.

In order to apply the thermal diagram to gas-engines, it is necessary to know the specific heats of the mixture used. The relative weights must be known of exhaust products, fuel charge, and air, also the chemical composition of both fuel and exhaust. The specific heats of each part multiplied by the percentage of weight it represents gives its relative specific heat, the total specific heat being the sum of these individual relative heats.